

## Preamble

Asymmetric Plain Methods are as liable to internal falseness as Treble-dodging methods. In the example of Bishopthorpe Bob Minor, falseness is generated by the rows with the treble in 5ths. These are 426513 and 423516. By the usual methods, it is obvious that the lead starting from 126453 is false against the first lead of the method. (For Bishopthorpe Bob, this is the only falseness).

For an extent, we cannot ring the lead starting 126453 BUT this has the lead-end 143256 and we MUST include this in the extent so we MUST ring the lead starting from 143256. This is a *Required Lead Head* of the method.

Now, there are several ways of relating 426513 and 423516 but Smith's Theorem uses the transposition from one to the other. If  $426513 = A$ , then  $423516 = TA$ , in an obvious notation (where  $T = 126453$ ).

## Proof

The first lead contains the (ordered pair of) rows  $\{A, TA\}$ .

There exists another lead which has  $TA$  in the 1<sup>st</sup> position and is  $\{TA, T^2A\}$ . Now, if  $T^2 = e$ , this is just the statement that a lead rung backwards in a symmetric method is false. Whether or not, this lead cannot be rung since we have already used  $TA$  in the 1<sup>st</sup> lead.

Suppose now  $T^2 \triangleleft e$ . We still have to include row  $T^2A$  but this must be at start of the lead so the lead must contain  $\{T^2A, T^3A\}$  and this is a *required lead*.

In short, the leads of the extent must be of the form  $\{A, TA\}$ ,  $\{T^2A, T^3A\}$ ,  $\{T^4A, T^5A\}$ ,... and the false leads are  $\{TA, T^2A\}$ ,  $\{\{T^2A, T^3A\}, \dots$

Now, if  $T$  is to an odd power, three, say, the second required lead is  $\{T^2A, A\}$  and this contains a row,  $A$ , from the first lead.

This completes the proof.

This may be more easily seen in a diagram:

<b>A</b>	<i>TA</i>	<b>T<sup>2</sup>A</b>	<i>T<sup>3</sup>A</i>	<b>T<sup>4</sup>A</b>	etc.
<b>TA</b>	<i>T<sup>2</sup>A</i>	<b>T<sup>3</sup>A</b>	<i>T<sup>4</sup>A</i>	<b>T<sup>5</sup>A</b>	

The leads in italics are the trivially false ones

## Example

123456  
214365 x  
241356 4  
423165 x  
243615 3  
426351 x  
462531 1  
645213 x  
654123 1  
561432 x  
516342 1  
153624 x

The first lead of Yorkshire Court B6 is given. By inspection, we see that there is symmetry when the treble is in every place except 3rds and 4ths so we concentrate on these two.

The 3rds place rows are 241356 and 561432. From the above, in an obvious notation,  $T_3 = 563241$  which is of order five, i.e., odd. The false lead deriving from this is 165324 and the required lead-head (RLH) is 125463 – the false lead rung backward.

The 4ths place rows are 423165 and 654123 for which  $T_4 = 561423$ , also of order five. (The false lead generated is the same as is the required

lead.)

The first RLH generates a group of which the third element is 126435.

Following the proof, we have to ring leads which contain  $T^2A$  and  $T^4A$  as the first occurrence of 3rds place. Given the theory, we are interested in  $T^4A$  which is 631524. Pricking the changes of the lead following this row does indeed give 241356 which is the third row of the first lead. QED

In short – we have to ring even-powered transpositions in the top half of the lead and odd-powered in the bottom half. If the order of the transposition is odd, a row we have already rung appears in the bottom half of a lead.